REPORT No. 807

A METHOD OF ANALYSIS OF V–G RECORDS FROM TRANSPORT OPERATIONS

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SUMMARY

A method has been developed for interpreting V–G records taken during the course of commercial transport operation. This method involves the utilization of fairly simple statistical procedures to obtain "flight envelopes," which predict that, on the average, in a stated number of flight hours, one value of airspeed will exceed the envelope, and one positive and one negative acceleration increment will exceed the envelope with equal probability of being experienced at any airspeed. Comparison with the actual data obtained from various airplanes and from various airlines indicates that these envelopes predict the occurrences of large values of acceleration and airspeed with a high degree of accuracy.

INTRODUCTION

Acceleration and airspeed data have been collected at the Langley Laboratory of the NACA in the form of V–G records (reference 1) that represent about 200,000 hours of flight with various transport airplanes. These records, which have been supplied by the airlines, together with occasional supplementary information concerning atmospheric conditions and unusual operating practices, constitute almost all the data on applied loads that are available from commercial transport operations

The V–G record presents the envelope of indicated airspeed against vertical acceleration for the length of time that it is installed in the airplane. The nature of this envelope is such that only peak values of acceleration may be discerned, so that, for a given amount of flight time, only the largest accelerations experienced at each speed can be evaluated. Since little is known about the actual conditions under which the data were obtained, average conditions of wing loading, load distributions, and operating practices must be assumed. Despite the apparent limitations of such data, the V–G data have the advantage of representing a wide variety of operating conditions and of presenting the large values of acceleration that are significant in the investigation of the gust-load requirements. Thus, analysis of the V–G records may be expected to be of value in the evaluation of present requirements or in pointing the way for future modifications.

A rough check of present requirements may be made by comparing the composite of all the V–G envelopes from a given airplane with the accelerations for which the airplane is designed. Composite envelopes for the various airplanes with which V–G data have been obtained are presented in reference 2. It is evident, however, that the composite envelopes, although of value in the presentation of the overall picture of airspeed and acceleration, contribute little to the determination of trends in the data or to the prediction of the acceleration experiences of transport airplanes.

A growing appreciation of the problem of life expectancy of aircraft has indicated the need for a method of analysis of the V–G data that would permit predictions concerning the number and magnitude of the large accelerations which would be experienced in a given number of hours of normal transport operation. The object of the present report is to describe such a method and to show how it may be applied to the V–G data from any transport airplane. This method involves the utilization of fairly simple statistical procedures to arrive at "flight envelopes" which predict that, on the average, in a given number of flight hours, one value of airspeed will exceed the envelope and one positive and one negative acceleration increment will exceed the envelope with equal probability of being experienced at any airspeed.

SYMBOLS

τ	average flight time per record, hours
T	total flight time, hours
V_{max}	maximum indicated airspeed on V–G record, miles per hour
P_s	probability that maximum indicated airspeed on V–G record will exceed a given value
V_T	indicated airspeed that will be exceeded, on the average, once in T hours, miles per hour
k	number of 10-mile-per-hour airspeed intervals into which the speed range of airplane is divided
Δn_{max}	maximum positive or negative acceleration increment on V–G record, g units
$P_{\Delta n}$	probability that maximum acceleration increment on V–G record will exceed a given value
Δn_T	acceleration increment that will be exceeded, on the average, once in kT hours in a given speed range, g units

V_0	indicated airspeed at which maximum positive or negative acceleration increment on V–G record is experienced, miles per hour
P_0	probability that V_0 will occur in a given speed range
ΣP_0	probability that V_0 will exceed a given value
P	probability that a given value of acceleration will be exceeded in a given speed range
$\overline{V}_{max}$, $\overline{\Delta n}_{max}$, $\overline{V}_0$	average values of distributions of V_{max}, Δn_{max}, and V_0, respectively
σ_v, $\sigma_{\Delta n}$, σ_0	standard deviations of distributions of V_{max}, Δn_{max}, and V_0, respectively
α_v, $\alpha_{\Delta n}$, α_0	coefficients of skewness of distributions of V_{max}, Δn_{max}, and V_0, respectively

The term "probability" used herein may be interpreted as the ratio of the number of records on which a given event occurs to the total number of records. All airspeeds mentioned are indicated airspeeds.

METHOD

The method presented may be used to predict the acceleration experiences of transport airplanes from available V–G records. Briefly, the steps in the procedure are the selection from the basic data of the records suitable for analysis, the determination from these records of the probability of occurrence of large values of airspeed and acceleration, and the combination of these probabilities to obtain flight envelopes.

Basic data.—Because of the desirability of a uniform number of flight hours per record, only the records that have a reasonably constant number of flight hours should be used. The wide variation in the number of flight hours of available records, however, requires that fairly broad limits be allowed in order to obtain a sufficient number of records for the analysis. No definite limits can be set but, on the basis of the work done to date, it appears that a range of about 30 percent of the total variation in flight hours may be allowed.

Six values are taken from each of the records to be analyzed: the flight time, the maximum indicated airspeed V_{max}, the maximum positive and maximum negative acceleration increments Δn_{max}, and the indicated airspeeds V_0 at which the maximum acceleration increments are experienced.

Probability distributions.—The probability distributions of V_{max}, Δn_{max}, and V_0 are required in the analysis. It has been found that Pearson Type III probability curves (reference 3) can be used to determine the probabilities P_v, $P_{\Delta n}$, and ΣP_0 that V_{max}, Δn_{max}, and V_0, respectively, will exceed given values. With the assumption that the distributions of positive and negative acceleration increments are identical, the values of Δn_{max} are combined without regard to sign in the determination of $P_{\Delta n}$.

The Pearson Type III curves form a three-parameter family; the parameters for a particular distribution are determined from the mean value, the standard deviation, and the coefficient of skewness of the distribution. The actual computation of these curves is somewhat involved and depends on tables that are not in common use. For convenience, the curves having parameters within the range that might be expected in the analysis of V–G data are presented in figure 1. The abscissa t in this figure is the so-called standard statistical scale (reference 3) so that, for the distribution of V_{max},

$$t=\frac{V_{max}-\overline{V}_{max}}{\sigma_v} \tag{1a}$$

for the distribution of Δn_{max},

$$t=\frac{\Delta n_{max}-\overline{\Delta n}_{max}}{\sigma_{\Delta n}} \tag{1b}$$

and, for the distribution of V_0,

$$t=\frac{V_0-\overline{V}_0}{\sigma_0} \tag{1c}$$

The mean value, the standard deviation, and the coefficient of skewness of the distribution of V_{max} may be determined from the following formulas:

$$\overline{V}_{max}=\frac{1}{N}\Sigma V_{max}$$

$$\sigma_v=\sqrt{\frac{1}{N}\Sigma(V_{max}-\overline{V}_{max})^2}$$

$$\alpha_v=\frac{1}{N\sigma_v^3}\Sigma(V_{max}-\overline{V}_{max})^3$$

where N represents the number of observations upon which the distribution is based. Similar formulas hold for the distributions of Δn_{max} and V_0. In the use of figure 1 to construct the probability curves, values of t corresponding to selected values of V_{max}, Δn_{max}, or V_0 are computed and the probabilities are then determined from the curve in figure 1 that corresponds to the proper value of α. Linear interpolation for values of α not shown in figure 1 is adequate.

In the subsequent analysis, the probability ΣP_0 is assumed to give a reliable representation of the probability that the airspeed at which a given large acceleration is experienced will exceed a given value. Inspection of the data from various airplanes, however, has shown that this assumption is valid only up to the normal cruising speed of the airplane, since, although large accelerations are frequently experienced at high speeds, the maximum accelerations on a record are seldom found beyond this point. Accordingly, it has been found necessary to modify the Type III curve for the distribution of V_0 so as to present a suitable estimate of the probability that the speed at which a given large acceleration is experienced will exceed a given value. Experimentation has shown that the necessary modification can be conveniently effected by extrapolating to the high values of airspeed by means of an exponential curve (a straight line on semilog paper) drawn tangent to the Type III curve at the normal cruising speed of the airplane. Values of ΣP_0 at high values of airspeed are taken from this exponential curve.

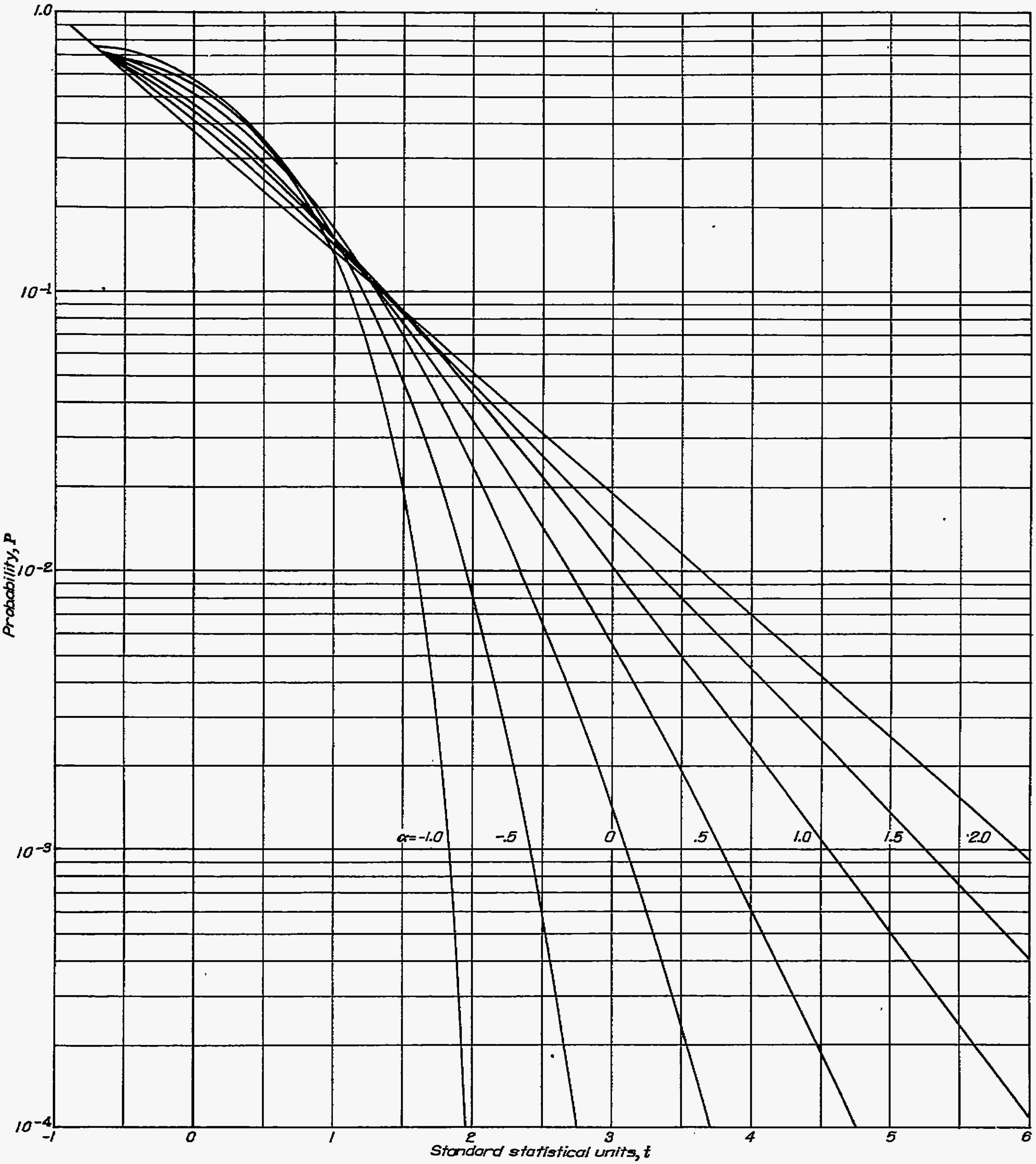

FIGURE 1.—Pearson Type III probability curves.

The flight envelope for a given number of hours is determined by the number of accelerations that will exceed given values in different speed ranges. Thus, construction of the envelope depends upon a knowledge of the probability P that a given large value of acceleration will be exceeded in a given speed range. With the assumptions that

(1) Acceleration and airspeed are statistically independent (that is, if the airplane were flown for equal periods of time at all speeds, the probability of exceeding a given value of acceleration would be the same at all airspeeds)

(2) The distribution of Δn_{max} gives a reliable representation of the distribution of large accelerations

(3) The modified distribution of V_0 gives a reliable representation of the velocities at which all large accelerations occur

the probability P may be expressed as the product of two known probabilities; thus

$$P=P_{\Delta n}P_0 \tag{2}$$

where P_0 is the probability that V_0 will occur in the given speed range.

Some explanation of assumption (1) is in order. In the strictest sense acceleration and airspeed are not statistically independent since, if the airplane were flown at random at different speeds, the highest accelerations would be experienced at the highest speeds. Transport airplanes, however, are not flown at random speeds; in fact, the speed is usually determined by weather conditions so that the airplane flies at high speeds in smooth air and at reduced speeds in turbulent air. Thus, the assumption of statistical independence is to be interpreted herein as meaning that if the airplane were flown for equal periods of time at all speeds under conditions at which these speeds are normally attained then the probability of exceeding a given value of acceleration would be the same at all airspeeds.

Envelope airspeed.—The first step in the construction of the flight envelope representing, say, T hours of flight is the determination of the airspeed V_T that will be exceeded once in T hours. If τ is the average flight time per record, the desired airspeed is the one that is exceeded, on the average, once in T/τ records. Thus, V_T is determined from the equation

$$P_v=\frac{\tau}{T} \tag{3}$$

and from the probability curve for P_v.

Envelope accelerations.—It has been found convenient to divide the expected speed range of the airplane into 10-mile-per-hour intervals. If the minimum airspeed for transport airplanes (except during landing and take-off) is assumed to be 100 miles per hour, in the flight envelope for T hours,

$$k=\frac{V_T-100}{10} \tag{4}$$

speed intervals must be considered. (The nearest integral value of k is satisfactory.)

If one acceleration is to exceed the flight envelope in T hours with equal probability of being experienced in each of the k speed ranges, then, on the average, one acceleration in kT hours should exceed the flight envelope in each speed range; that is, on the average, one in kT/τ records should have an acceleration exceeding the envelope in a given speed range. Thus, $P=\frac{\tau}{kT}$ so that with the use of equation (2), the acceleration increment Δn_T, which determines the flight envelope for T hours in a given speed range, is found from the equation

$$P_{\Delta n}=\frac{\tau}{kTP_0} \tag{5}$$

and from the probability curve for $P_{\Delta n}$.

The values of V_T and Δn_T as determined in the preceding paragraphs are sufficient to construct the envelope representing T hours of flight. Positive and negative values of Δn_T are plotted at the midpoints of the corresponding speed ranges and a smooth curve is drawn through the plotted points. The envelope is closed at the high-speed end by a straight line through V_T perpendicular to the airspeed axis.

In the present analysis, no attempt is made to classify the acceleration peaks taken from the records as due to gusts, maneuvers, or gust maneuvers. Such a classification is at best highly uncertain. Inasmuch as experience indicates that most of the large loads imposed during normal transport operation are due to gusts, the assumption that the flight envelopes of this report represent gust conditions seems reasonable. Thus, these envelopes may be converted to gust-velocity envelopes by means of the sharp-edged-gust formula (reference 1).

EXAMPLE

In order to illustrate the foregoing method of analysis, the actual computations necessary for the construction of a particular flight envelope are presented. The data were obtained from DC-3 airplanes of a particular airline during the period from 1937 to 1941. The results given herein cannot be applied generally since they represent only a small part of the available data.

Basic data.—The data available for analysis consisted of 35 records totaling 17,675 hours of flight. Because of the wide range of flight hours, however, only records that represented between 575 and 800 hours were used. There were 15 such records, totaling 9691 hours of flight and having an average record time τ of about 650 hours. The values of V_{max}, Δn_{max}, and V_0 read from these records are summarized in table I in the form of frequency distributions.

TABLE I

FREQUENCY DISTRIBUTIONS OF V_{max}, Δn_{max}, AND V_0 FROM V-G RECORDS FROM DC-3 AIRPLANES

V_{max} distribution		Δn_{max} distribution		V_0 distribution	
V_{max} (mph)	Frequency	Δn_{max} (g units)	Frequency	V_0 (mph)	Frequency
215–219	1	0.60–0.69	1	100–109	0
220–224	4	0.70–0.79	1	110–119	1
225–229	3	0.80–0.89	2	120–129	0
230–234	4	0.90–0.99	2	130–139	1
235–239	2	1.00–1.09	4	140–149	3
240–244	0	1.10–1.19	4	150–159	3
245–249	0	1.20–1.29	6	160–169	4
250–254	1	1.30–1.39	4	170–179	6
		1.40–1.49	2	180–189	7
		1.50–1.59	0	190–199	2
		1.60–1.69	0	200–209	3
		1.70–1.79	3		
		1.80–1.89	0		
		1.90–1.99	1		
$\overline{V}_{max}=229.84$ $\sigma_v=8.34$ $\alpha_v=1.05$		$\overline{\Delta n}_{max}=1.23$ $\sigma_{\Delta n}=0.30$ $\alpha_{\Delta n}=0.46$		$\overline{V}_0=172.00$ $\sigma_0=21.16$ $\alpha_0=-0.57$	

Probability distributions.—The average values, the standard deviations, and the coefficients of skewness of the distributions of V_{max}, Δn_{max}, and V_0 have been computed by standard methods (reference 3) and the results are presented in table I. The Pearson Type III probability curves determined from these computed values and the curves in figure 1 are shown in figures 2 to 4 together with the cumulative frequency distributions obtained from table I. (The cumulative frequency distributions present the relative frequencies, obtained from the basic data, of airspeeds and accelerations that exceed given values.) Also shown in figure 4 is the added "straight-line" extrapolation of the V_0 probability curve at the cruising speed of the DC-3 airplane, which is 180 miles per hour.

In order to illustrate the manner in which the Type III curves are constructed, one point on the Δn_{max} probability curve will be determined. From equation (1b) and table I,

$$t=\frac{\Delta n_{max}-1.23}{0.30}$$

For $\Delta n_{max}=2.0$, $t=2.56$, and from figure 1 for $\alpha_{\Delta n}=0.46$, the probability is found to be 0.011. This value is plotted in figure 3 for $\Delta n_{max}=2.0$.

Figures 2 and 3 give the probability that selected values of airspeed or acceleration increment will be exceeded. For example, from figure 2, the probability that V_{max} will exceed 240 miles per hour is 0.12 so that, on the average, 1 in about 8.3 records will have a maximum airspeed greater than 240 miles per hour. Since $\tau=650$ for these data, it may be concluded that this value of airspeed will be exceeded, on the average, once in every 5400 hours of flight.

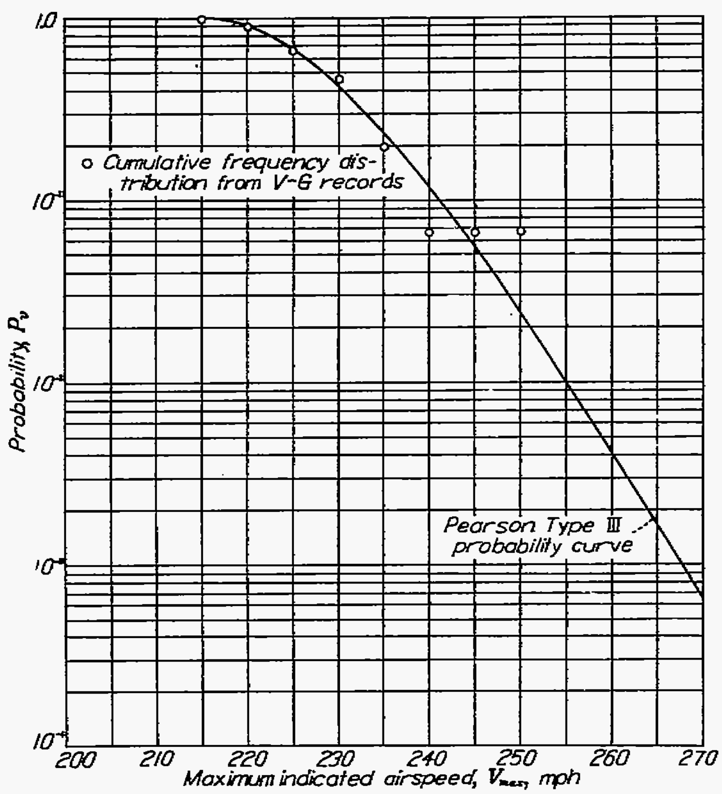

FIGURE 2.—Probability that maximum indicated airspeed on a record will exceed a given value.

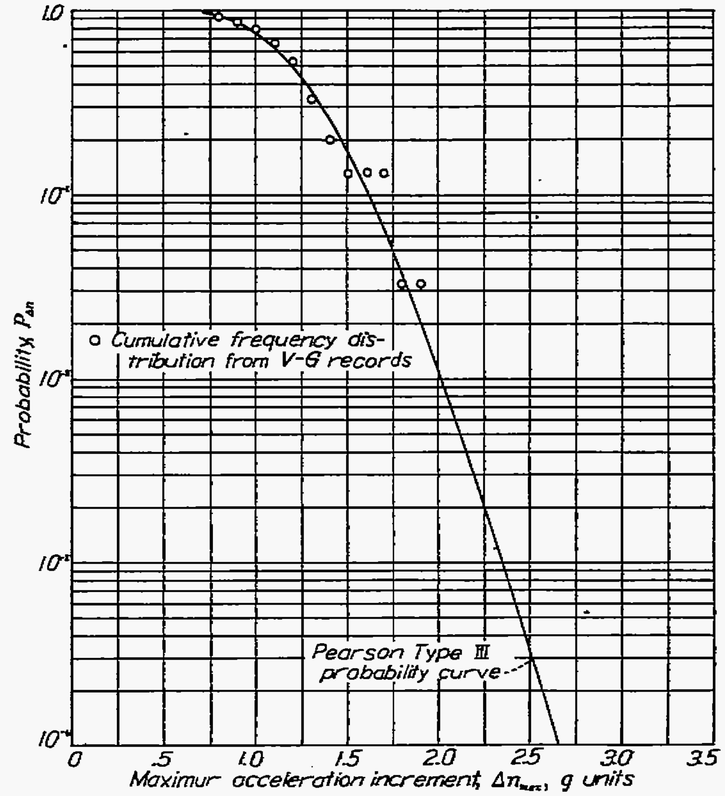

FIGURE 3.—Probability that the maximum acceleration increment on a record will exceed a given value.

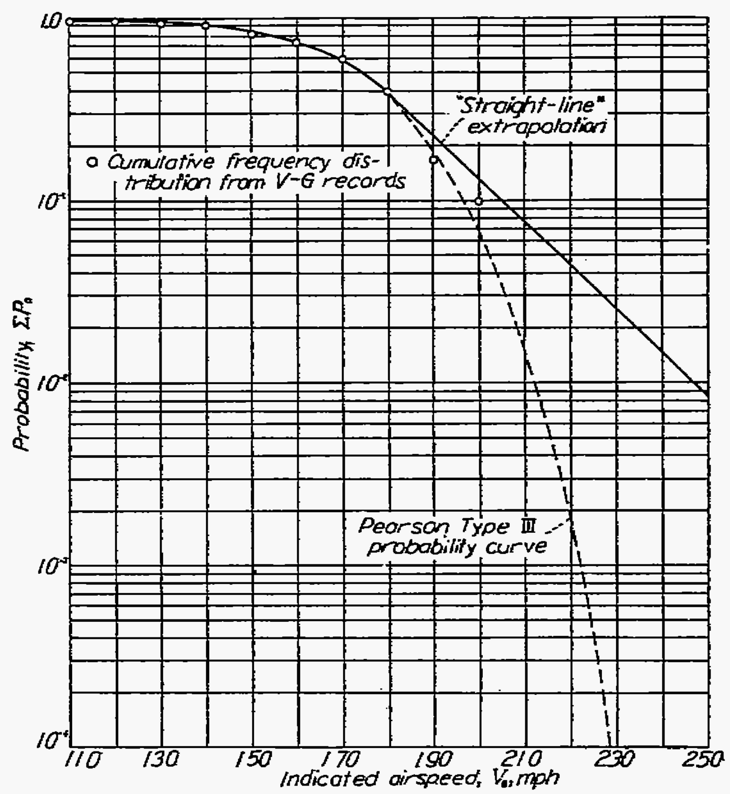

FIGURE 4.—Probability that indicated airspeed at which maximum acceleration increment is experienced will exceed a given value.

Values of P_0 are obtained by subtracting successive values of ΣP_0 taken from figure 4. For example, the probability that the maximum acceleration will occur in the speed interval from 180 to 190 miles per hour is 0.38−0.22 or 0.16.

Envelope airspeed.—The construction of the flight envelope is now dependent only upon the choice of the number of flight hours T. In this example, the value of 10,000 hours has been selected for T. This choice is quite arbitrary, and only slight modifications in the numerical work are necessary to determine the envelope that corresponds to any desired value of T. With the use of equation (3),

$$P_v=\frac{650}{10000}$$
$$=0.0650$$

so that, from figure 2, V_T=244 miles per hour.

Envelope accelerations.—Since V_T=244 miles per hour, equation (4) shows that k=14. Thus, equation (5) becomes

$$P_{\Delta n}=\frac{650}{(14)(10000)P_0}$$
$$=\frac{0.00463}{P_0}$$

TABLE II

SUMMARY OF CALCULATIONS NECESSARY FOR THE DETERMINATION OF THE 10,000-HOUR FLIGHT ENVELOPE FOR DC-3 AIRPLANES

$$\left[P_{\Delta n}=\frac{\tau}{kTP_0}=\frac{0.00463}{P_0}\right]$$

V_0	ΣP_0	P_0	$P_{\Delta n}$	Δn_T
100	1.00			
		0.01	0.463	1.23
110	.99			
		.01	.463	1.23
120	.98			
		.02	.232	1.43
130	.96			
		.03	.154	1.53
140	.93			
		.08	.0579	1.73
150	.85			
		.11	.0421	1.78
160	.74			
		.16	.0289	1.85
170	.58			
		.20	.0232	1.88
180	.38			
		.16	.0289	1.85
190	.220			
		.092	.0503	1.75
200	.128			
		.054	.0857	1.65
210	.074			
		.031	.149	1.53
220	.043			
		.019	.244	1.42
230	.024			
		.010	.463	1.23
240	.014			

Table II shows how the values of $P_{\Delta n}$ for the various speed ranges may be conveniently obtained. Values of ΣP_0 are taken from figure 4 and P_0 is obtained by subtracting successive values of ΣP_0. The values of Δn_T are obtained from figure 3.

The values obtained for V_T and Δn_T are plotted in figure 5 and smooth curves are drawn through the plotted values of Δn_T. The flight envelope for 10,000 hours is thus fully determined.

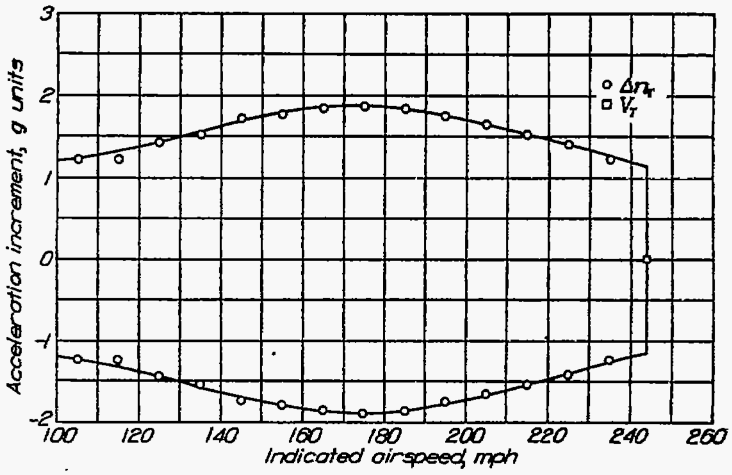

FIGURE 5.—Construction of flight envelope corresponding to 10,000 hours of flight with DC-3 airplanes.

DISCUSSION

The flight envelopes obtained by the methods of the present report predict that, on the average, in a stated number of hours of flight the maximum value of airspeed will be exceeded once and that one positive and one negative acceleration increment will exceed the envelope with equal probability of being experienced at any airspeed. The determination of these envelopes depends only upon the selection of the maximum values of airspeed and acceleration from the V-G records and involves what seems to be a minimum of computational work. Of course, similar envelopes could be obtained by the straightforward but highly laborious method of dividing the speed range of the airplane into, say, 10-mile-per-hour ranges and finding the distributions of maximum acceleration increments taken from each of these speed ranges. In order to compare the two methods, this procedure has been applied to the data from the DC-3 airplane used in the present report and the results for 10,000 hours of flight are compared in figure 6 with the 10,000-hour flight envelope of figure 5. The two methods can be seen to yield similar results.

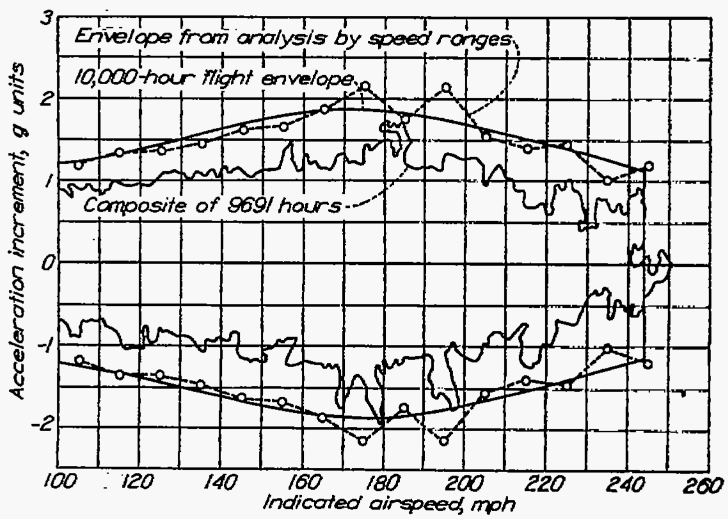

FIGURE 6.—Comparison of 10,000-hour flight envelope and envelope obtained by separate analysis of each speed range. Actual composite of records on which the envelopes were based.

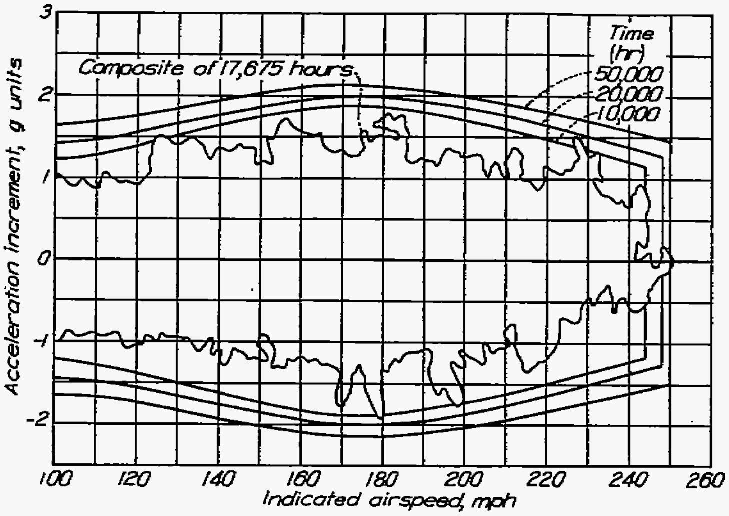

FIGURE 7.—Comparision of calculated flight envelopes with the composite of the V–G envelopes from DC–3 airplanes.

The composite of the 15 V–G records representing 9691 hours of flight, which have been used in determining the flight envelopes for the DC–3, is shown in figure 6. It may be noted that two airspeeds and two negative acceleration increments exceed the 10,000-hour envelope.

The fact that only 15 of the available 35 V–G records were used in the analysis affords an opportunity to investigate the capacity of the flight envelopes to predict acceleration and airspeed experiences. The composite of all the 35 records representing 17,675 hours of flight is shown in figure 7, together with the 10,000-hour, 20,000-hour, and 50,000-hour flight envelopes determined from the 15 selected records. Interpolation for about 17,500 hours shows that one positive acceleration increment and one airspeed exceed the envelope. The fact that exactly the right number of positive acceleration increments and airspeeds exceeded this one envelope is not particularly important, and the fact that no negative acceleration increments exceeded the envelope is equally unimportant since predictions based on a statistical analysis represent average conditions and can be verified only by the examination of large masses of data. Since eight sets of data, involving four different airplanes and four airlines, were available, a check of the present method can be made by comparing the calculated envelopes with the corresponding V–G composites. For the eight envelopes, nine positive acceleration increments, ten negative acceleration increments, and seven airspeeds were found to exceed the envelopes. Thus, the flight envelopes obtained by the methods of the present report appear to predict the occurrences of large values of acceleration and airspeed with a high degree of accuracy.

In the application of these methods to a particular set of V–G data, it appears that satisfactory results may be obtained with a minimum of about 15 records representing not less than about 2500 flight hours. The actual number of flight hours per record is not particularly important, provided the range of flight hours is not too large. Probability curves were derived from sets of data with different average record times and were found to be essentially similar; a change in scale made the curves comparable with regard to the number of hours required to exceed any given acceleration. Thus, identical results were obtained from two sets of records from the same airplane and airline but with widely different flight hours per record.

The variations in the number of flight hours on the records accumulated to date are apparently due to practical limitations that affect the installation and removal of the records from the airplanes. In order to increase the accuracy of the present type of analysis, the means of collecting V–G data should be improved so that more uniform flight hours per record may be assured in the future.

CONCLUDING REMARKS

Statistical methods of analysis of V–G records are developed to obtain "flight envelopes" for transport planes. These envelopes predict that, on the average, in a given number of flight hours, one value of airspeed will exceed the envelope and one positive and one negative acceleration increment will exceed the envelope with equal probability of being experienced at any airspeed.

Comparisons of the flight envelopes with the composite V–G envelopes indicated that these envelopes predict the occurrences of large values of accelerations and airspeeds with a high degree of accuracy.

In order to obtain satisfactory results, a minimum of about 15 records representing not less than about 2500 flight hours should be used in the analysis.

The number of flight hours per record is not particularly important in the analysis provided the range of flight hours in the set of records used is not too large.

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LANGLEY FIELD, VA., *August 31, 1945.*

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